

NAG Fortran Library Routine Document

F08UNF (ZHBGV)

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of *bold italicised* terms and other implementation-dependent details.

1 Purpose

F08UNF (ZHBGV) computes all the eigenvalues and, optionally, the eigenvectors of a complex generalized Hermitian-definite banded eigenproblem, of the form

$$Az = \lambda Bz,$$

where A and B are Hermitian and banded, and B is also positive-definite.

2 Specification

```

SUBROUTINE F08UNF (JOBZ, UPLO, N, KA, KB, AB, LDAB, BB, LDBB, W, Z, LDZ,
1                WORK, RWORK, INFO)
    INTEGER          N, KA, KB, LDAB, LDBB, LDZ, INFO
    double precision W(*), RWORK(*)
    complex*16      AB(LDAB,*), BB(LDBB,*), Z(LDZ,*), WORK(*)
    CHARACTER*1     JOBZ, UPLO

```

The routine may be called by its LAPACK name *zhbgv*.

3 Description

The generalized Hermitian-definite band problem

$$Az = \lambda Bz$$

is first reduced to a standard band Hermitian problem

$$Cx = \lambda x,$$

where C is a Hermitian band matrix, using Wilkinson's modification to Crawford's algorithm (see Crawford (1973) and Wilkinson (1977)). The Hermitian eigenvalue problem is then solved for the eigenvalues and the eigenvectors, if required, which are then backtransformed to the eigenvectors of the original problem.

The eigenvectors are normalized so that the matrix of eigenvectors, Z , satisfies

$$Z^H A Z = \Lambda \quad \text{and} \quad Z^H B Z = I,$$

where Λ is the diagonal matrix whose diagonal elements are the eigenvalues.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

Crawford C R (1973) Reduction of a band-symmetric generalized eigenvalue problem *Comm. ACM* **16** 41–44

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Wilkinson J H (1977) Some recent advances in numerical linear algebra *The State of the Art in Numerical Analysis* (ed D A H Jacobs) Academic Press

5 Parameters

- 1: JOBZ – CHARACTER*1 *Input*
On entry: if JOBZ = 'N', compute eigenvalues only.
 If JOBZ = 'V', compute eigenvalues and eigenvectors.
Constraint: JOBZ = 'N' or 'V'.
- 2: UPLO – CHARACTER*1 *Input*
On entry: if UPLO = 'U', the upper triangles of A and B are stored.
 If UPLO = 'L', the lower triangles of A and B are stored.
- 3: N – INTEGER *Input*
On entry: n , the order of the matrices A and B .
Constraint: $N \geq 0$.
- 4: KA – INTEGER *Input*
On entry: ka , the number of superdiagonals of the matrix A if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'.
Constraint: $KA \geq 0$.
- 5: KB – INTEGER *Input*
On entry: kb , the number of superdiagonals of the matrix B if UPLO = 'U', or the number of subdiagonals if UPLO = 'L'.
Constraint: $KB \geq 0$.
- 6: AB(LDAB,*) – **complex*16** array *Input/Output*
Note: the second dimension of the array AB must be at least $\max(1, N)$.
On entry: the upper or lower triangle of the symmetric band matrix A , stored in the first $ka + 1$ rows of the array. The j th column of A is stored in the j th column of the array AB as follows:
 if UPLO = 'U', $AB(ka + 1 + i - j, j) = a_{ij}$ for $\max(1, j - ka) \leq i \leq j$;
 if UPLO = 'L', $AB(1 + i - j, j) = a_{ij}$ for $j \leq i \leq \min(n, j + ka)$.
On exit: the contents of AB are destroyed.
- 7: LDAB – INTEGER *Input*
On entry: the first dimension of the array AB as declared in the (sub)program from which F08UNF (ZHBGV) is called.
Constraint: $LDAB \geq KA + 1$.
- 8: BB(LDBB,*) – **complex*16** array *Input/Output*
Note: the second dimension of the array BB must be at least $\max(1, N)$.
On entry: the upper or lower triangle of the Hermitian band matrix B , stored in the first $kb + 1$ rows of the array. The j th column of B is stored in the j th column of the array BB as follows:
 if UPLO = 'U', $BB(kb + 1 + i - j, j) = b_{ij}$ for $\max(1, j - kb) \leq i \leq j$;
 if UPLO = 'L', $BB(1 + i - j, j) = b_{ij}$ for $j \leq i \leq \min(n, j + kb)$.
On exit: the factor S from the split Cholesky factorization $B = S^H S$, as returned by F08UTF (ZPBSTF).

- 9: LDBB – INTEGER *Input*
On entry: the first dimension of the array BB as declared in the (sub)program from which F08UNF (ZHBGV) is called.
Constraint: $LDBB \geq KB + 1$.
- 10: W(*) – *double precision* array *Output*
Note: the dimension of the array W must be at least $\max(1, N)$.
On exit: if INFO = 0, the eigenvalues in ascending order.
- 11: Z(LDZ,*) – *complex*16* array *Output*
Note: the second dimension of the array Z must be at least $\max(1, N)$.
On exit: if JOBZ = 'V', then if INFO = 0, Z contains the matrix Z of eigenvectors, with the *i*th column of Z holding the eigenvector associated with $W(i)$. The eigenvectors are normalized so that $Z^H B Z = I$.
 If JOBZ = 'N', Z is not referenced.
- 12: LDZ – INTEGER *Input*
On entry: the first dimension of the array Z as declared in the (sub)program from which F08UNF (ZHBGV) is called.
Constraints:
 if JOBZ = 'V', $LDZ \geq \max(1, N)$;
 $LDZ \geq 1$ otherwise.
- 13: WORK(*) – *complex*16* array *Workspace*
Note: the dimension of the array WORK must be at least $\max(1, N)$.
- 14: RWORK(*) – *double precision* array *Workspace*
Note: the dimension of the array RWORK must be at least $\max(1, 3 \times N)$.
- 15: INFO – INTEGER *Output*
On exit: INFO = 0 unless the routine detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the routine:

INFO < 0

If $INFO = -i$, the *i*th parameter had an illegal value. An explanatory message is output, and execution of the program is terminated.

INFO > 0

If $INFO = i$ and $i \leq N$, the algorithm failed to converge: *i* off-diagonal elements of an intermediate tridiagonal form did not converge to zero.

If $INFO = i$ and $i > N$, if $INFO = N + i$, for $1 \leq i \leq N$, then F08UTF (ZPBSTF) returned 'INFO = *i*: B is not positive-definite'. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

7 Accuracy

If B is ill-conditioned with respect to inversion, then the error bounds for the computed eigenvalues and vectors may be large, although when the diagonal elements of B differ widely in magnitude the eigenvalues and eigenvectors may be less sensitive than the condition of B would suggest. See Section 4.10 of Anderson *et al.* (1999) for details of the error bounds.

8 Further Comments

The total number of floating-point operations is proportional to n^3 if $\text{JOBZ} = 'V'$ and, assuming that $n \gg k_a$, is approximately proportional to $n^2 k_a$ otherwise.

The real analogue of this routine is F08UAF (DSBGV).

9 Example

This example finds all the eigenvalues of the generalized band Hermitian eigenproblem $Az = \lambda Bz$, where

$$A = \begin{pmatrix} -1.13 & 1.94 - 2.10i & -1.40 + 0.25i & 0 \\ 1.94 + 2.10i & -1.91 & -0.82 - 0.89i & -0.67 + 0.34i \\ -1.40 - 0.25i & -0.82 + 0.89i & -1.87 & -1.10 - 0.16i \\ 0 & -0.67 - 0.34i & -1.10 + 0.16i & 0.50 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 9.89 & 1.08 - 1.73i & 0 & 0 \\ 1.08 + 1.73i & 1.69 & -0.04 + 0.29i & 0 \\ 0 & -0.04 - 0.29i & 2.65 & -0.33 + 2.24i \\ 0 & 0 & -0.33 - 2.24i & 2.17 \end{pmatrix}.$$

9.1 Program Text

```
*      F08UNF Example Program Text
*      Mark 21. NAG Copyright 2004.
*      .. Parameters ..
INTEGER          NIN, NOUT
PARAMETER       (NIN=5,NOUT=6)
INTEGER          NMAX, KAMAX, KBMAX
PARAMETER       (NMAX=20,KAMAX=5,KBMAX=5)
INTEGER          LDAB, LDBB
PARAMETER       (LDAB=KAMAX+1,LDBB=KBMAX+1)
CHARACTER        UPLO
PARAMETER       (UPLO='U')
*      .. Local Scalars ..
INTEGER          I, INFO, J, KA, KB, N
*      .. Local Arrays ..
COMPLEX *16      AB(LDAB,NMAX), BB(LDBB,NMAX), DUMMY(1,1),
+               WORK(NMAX)
DOUBLE PRECISION RWORK(3*NMAX), W(NMAX)
*      .. External Subroutines ..
EXTERNAL         ZHBGV
*      .. Intrinsic Functions ..
INTRINSIC        MAX, MIN
*      .. Executable Statements ..
WRITE (NOUT,*) 'F08UNF Example Program Results'
WRITE (NOUT,*)
*      Skip heading in data file
READ (NIN,*)
READ (NIN,*) N, KA, KB
IF (N.LE.NMAX .AND. KA.LE.KAMAX .AND. KB.LE.KBMAX) THEN
*
*      Read the upper or lower triangular parts of the matrices A and
*      B from data file
*
      IF (UPLO.EQ.'U') THEN
        READ (NIN,*) ((AB(KA+1+I-J,J),J=I,MIN(N,I+KA)),I=1,N)
```

```

      READ (NIN,*) ((BB(KB+1+I-J,J),J=I,MIN(N,I+KB)),I=1,N)
ELSE IF (UPLO.EQ.'L') THEN
      READ (NIN,*) ((AB(1+I-J,J),J=MAX(1,I-KA),I),I=1,N)
      READ (NIN,*) ((BB(1+I-J,J),J=MAX(1,I-KB),I),I=1,N)
END IF
*
*   Solve the generalized Hermitian band eigenvalue problem
*   A*x = lambda*B*x
*
      CALL ZHBGV('No vectors',UPLO,N,KA,KB,AB,LDAB,BB,LDBB,W,DUMMY,1,
+           WORK,RWORK,INFO)
*
      IF (INFO.EQ.0) THEN
*
*       Print solution
*
          WRITE (NOUT,*) 'Eigenvalues'
          WRITE (NOUT,99999) (W(J),J=1,N)
ELSE IF (INFO.GT.N .AND. INFO.LE.2*N) THEN
          I = INFO - N
          WRITE (NOUT,99998) 'The leading minor of order ', I,
+           ' of B is not positive definite'
ELSE
          WRITE (NOUT,99997) 'Failure in ZHBGV. INFO =', INFO
END IF
ELSE
          WRITE (NOUT,*) 'NMAX too small'
END IF
STOP
*
99999 FORMAT (3X,(6F11.4))
99998 FORMAT (1X,A,I4,A)
99997 FORMAT (1X,A,I4)
END

```

9.2 Program Data

F08UNF Example Program Data

```

      4           2           1                               :Values of N, KA and KB

(-1.13, 0.00) ( 1.94,-2.10) (-1.40, 0.25)
              (-1.91, 0.00) (-0.82,-0.89) (-0.67, 0.34)
              (-1.87, 0.00) (-1.10,-0.16)
              ( 0.50, 0.00) :End of matrix A

( 9.89, 0.00) ( 1.08,-1.73)
              ( 1.69, 0.00) (-0.04, 0.29)
              ( 2.65, 0.00) (-0.33, 2.24)
              ( 2.17, 0.00) :End of matrix B

```

9.3 Program Results

F08UNF Example Program Results

```

Eigenvalues
-6.6089   -2.0416   0.1603   1.7712

```
